BAYESIAN ANALYSIS AND INTERPRETATION OF HEAVY-ION COLLISIONS

- Motivations & Goals
- Challenges & Methods
- Results & Interpretations

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Bayesian Parameter Determination

Method

S. Habib, K. Heitmann, D. Higdon, C. Nakhleh, B. Williams, PRD 76(2007) 083503
J.Novak,K. Novak,S. Pratt,J. Vredevoogd,C. Coleman-Smith, R. Wolpert, PRC 89 (2014) 034917

Heavy-Quark Diffusivity

Y.Xu,J.Bernhard, S.A.Bass, S.Cao, PRC 97 (2018) 014907

Initial State Parameterization

W.Ke, J.Scott Moreland, J.E. Bernhard, S.A.Bass, PRC 96 (2017) 044912
J.Bernhard, J.Scott Moreland, S.A. Bass, PRC 94 (2016) 024907
J.Scott Moreland, J.E. Bernhard, S.A. Bass, nucl-th 1808:0216
S.Pratt, E.Sangaline, P.Sorensen and H.Wang, PRL 114 (2015) 202301

Jet Energy Loss

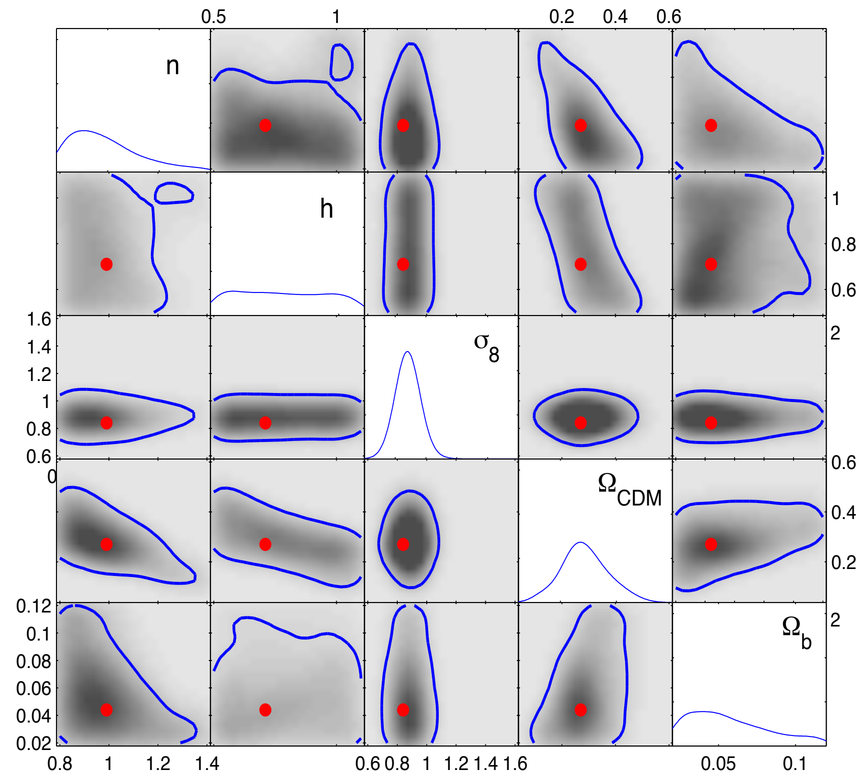
R.Soltz, JETSCAPE, Hard Probes Proc. (2019) DOI 10.22323/1/345.0048

Viscosity

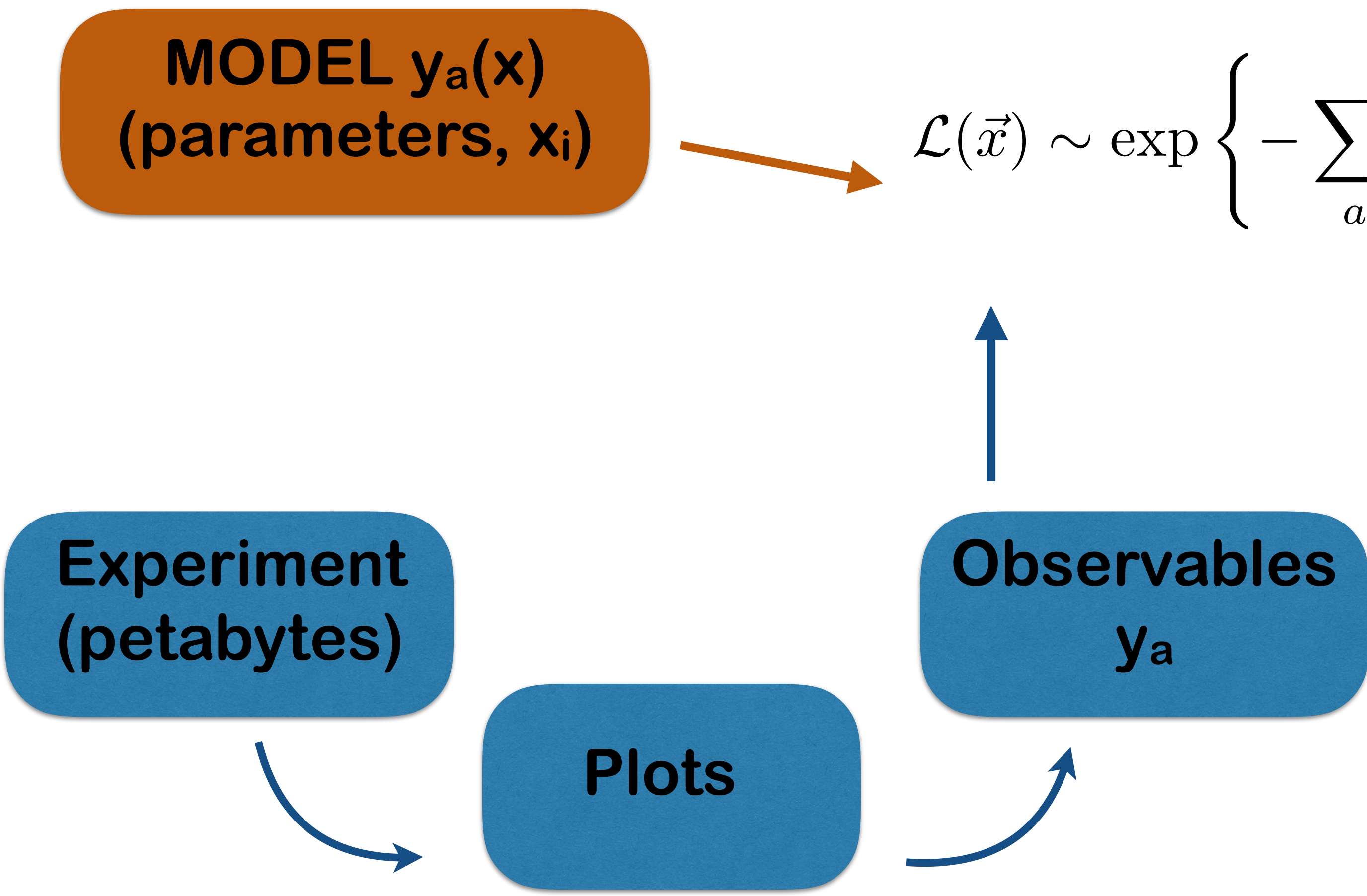
S.Pratt, E.Sangaline, P.Sorensen and H.Wang, PRL 114 (2015) 202301
J.Auvinen, J.E. Bernhard, S.A. Bass, I.Karpenko, PRC 97 (2018) 044905

Equation of State

S.Pratt, E.Sangaline, P.Sorensen and H.Wang, PRL 114 (2015) 202301



GOAL: Determine Likelihood

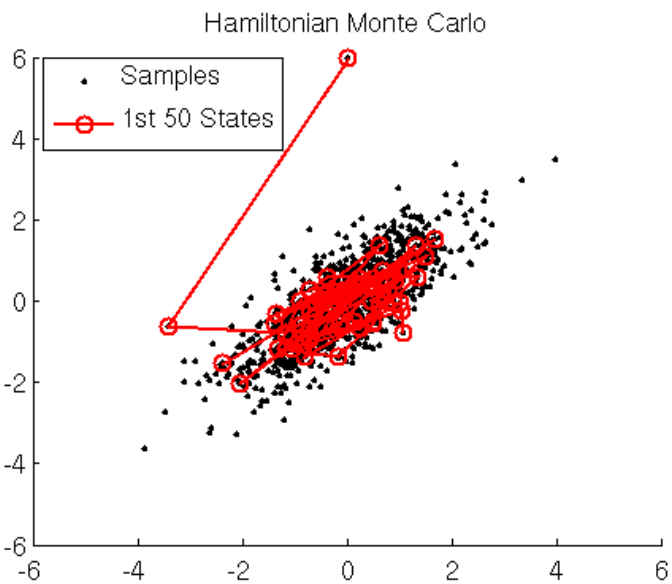


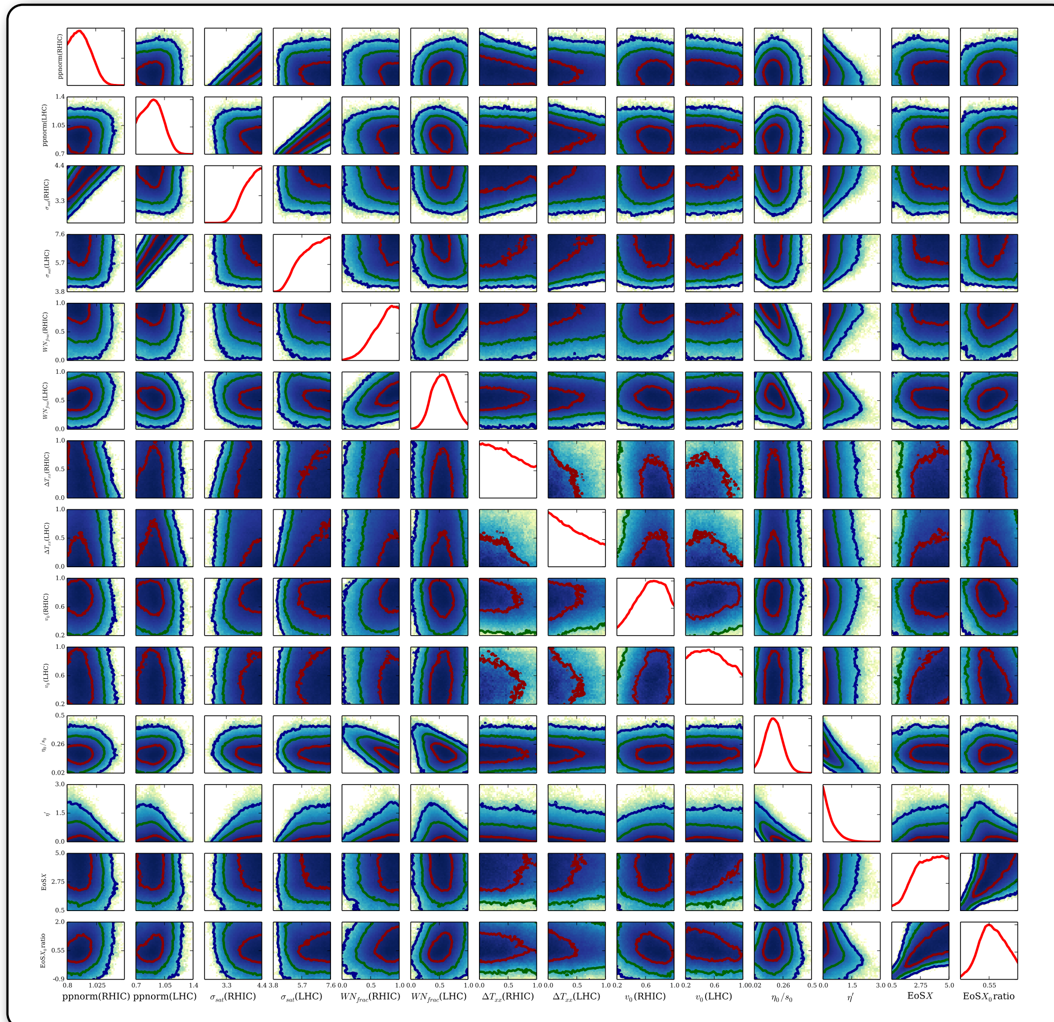
$$\mathcal{L}(\vec{x}) \sim \exp\left\{ -\sum_a \frac{(y_a^{(m)}(\vec{x}) - y_a)^2}{2\sigma_a^2} \right\}$$

GOAL: Determine Likelihood

$$\mathcal{L}(\vec{x}) \sim \exp\left\{-\sum_a \frac{(y_a^{(m)}(\vec{x}) - y_a)^2}{2\sigma_a^2}\right\}$$

Sample likelihood with MCMC





CHALLENGES

1. Expensive Model
2. Heterogenous Data
3. Expressing Uncertainties:
   — "systematic" model error (missing physics)
   — competing models (jet physics)
   — correlated errors (especially for theory)

$$\mathcal{L}(\vec{x}) \sim \exp\left\{-\sum_a \frac{(y_a^{(m)}(\vec{x}) - y_a)^2}{2\sigma_a^2}\right\}$$

→

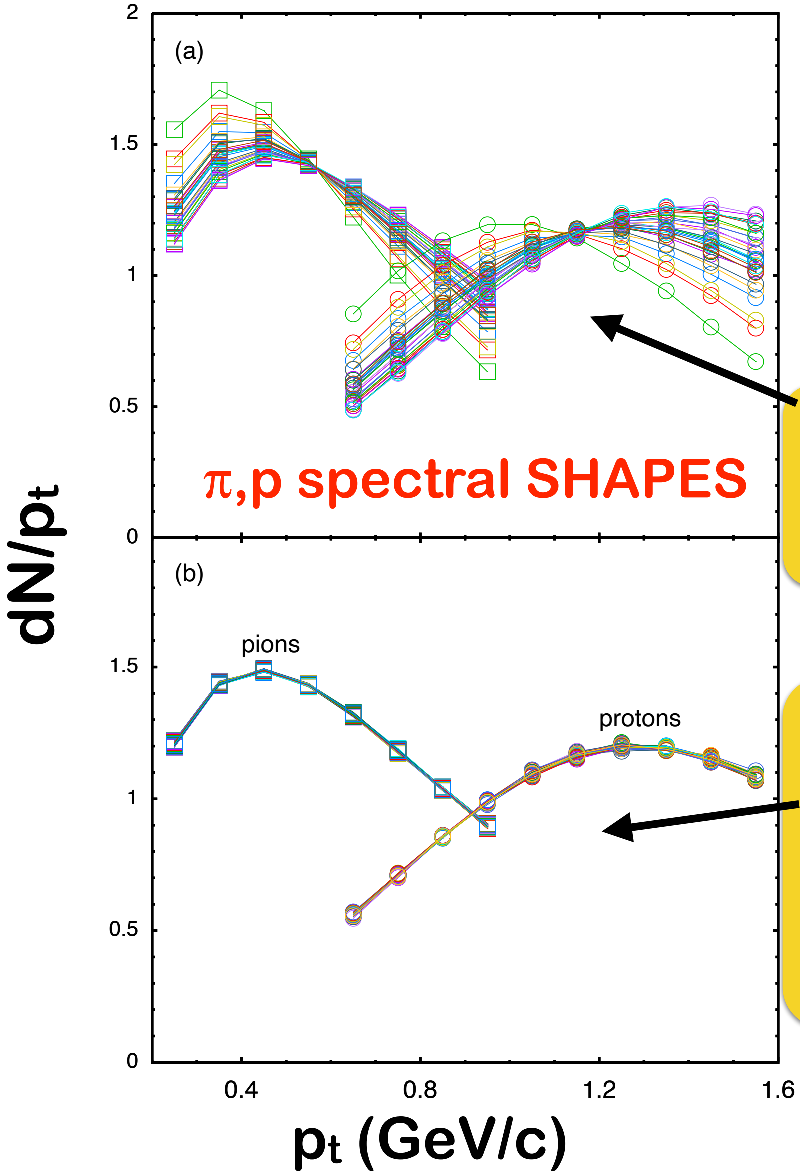
$$\mathcal{L}(\vec{x}) \sim \exp\left\{-\frac{1}{2}\sum_{ab}(y_a^{(m)}(\vec{x}) - y_a)\Sigma_{ab}^{-1}(y_b^{(m)}(\vec{x}) - y_b)\right\}$$

Distilling Heterogenous Data

1. Experiments reduce PBs to 100s of plots
2. Choose which data to analyze
   Does physics factorize?
3. Reduce each plot to a few values, y_a
   (use principle components)
4. Calculate global principal components, z_a
5. Resolving power of RHIC/LHC
   data reduced to $\lesssim$10 numbers!

Data Distillation

Spectral information encapsulated by two numbers, dN/dy & ⟨p_t⟩



model spectra from 30 random points in parameter prior

74 pion spectra: with 573<⟨p_t⟩$_\pi$< 575 MeV

44 proton spectra: with 1150<⟨p_t⟩$_p$< 1152 MeV

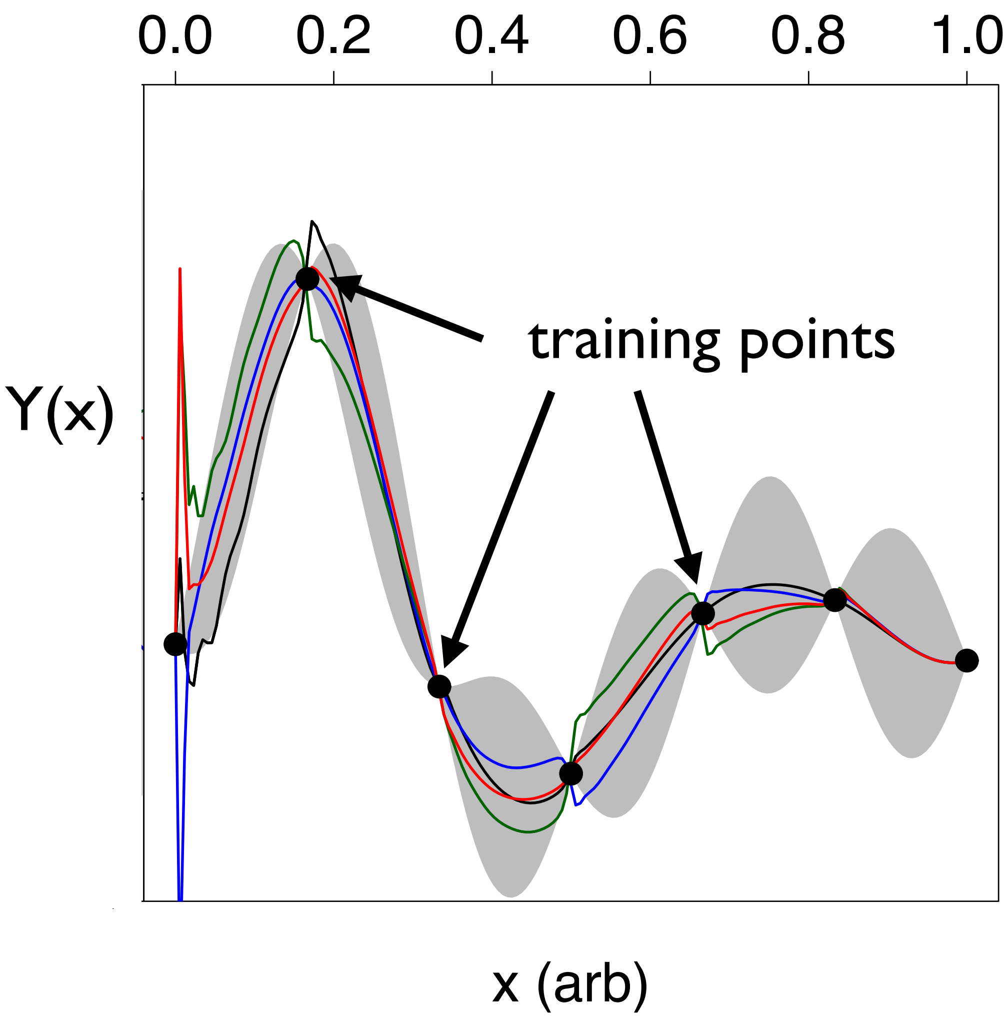
Correlated Uncertainties

1. Distill plots to small number of principal components★
2. Implement error matrix
3. “Nuisance” parameters

$$\frac{dN}{dp} = \frac{dN^{(\mathrm{m})}}{dp} + \alpha e^{-p/\lambda} \ldots$$

★applied here

Expensive Models



MCMC may need to repeat model millions of times — intractable

Gaussian Process Emulator

- Reproduces training points
- Assumes localized Gaussian covariance
- Must be trained, i.e. find "hyper parameters"
- Other methods also work

Results & Interpretation

To address these issues:

MADAI Collaboration
Models and Data Analysis Initiative
(active 2010-2017)

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY Duke UNIVERSITY THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL renci

1st MADAI Collaboration Meeting, SANDIA 2010

RHIC/LHC Global Analysis

S.Pratt, E.Sangaline, P.Sorensen and H.Wang, PRL 114 (2015) 202301

Parametric Initial State & Viscous Hydro & Hadron Cascade
14 Parameters (All for hydro)

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

RHIC Au+Au (100+100 GeV)
LHC Pb+Pb
30 Observables

- π,K,p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- v_2 Weighted by p_t

→ ***Likelihood*** ←

Initial State Parameters
(energy, WN vs. cgc, saturation, collective flow, SE tensor anisotropy)

$$\epsilon(\tau = 0.8\text{fm}/c) = f_{\text{wn}}\epsilon_{\text{wn}} + (1 - f_{\text{wn}})\epsilon_{\text{cgc}},$$

$$\epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_B\right)\right\} + (A \leftrightarrow B)$$

$$\epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_{\text{max}}\right)\right\}$$

$$T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\text{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

Equation of State and Viscosity

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

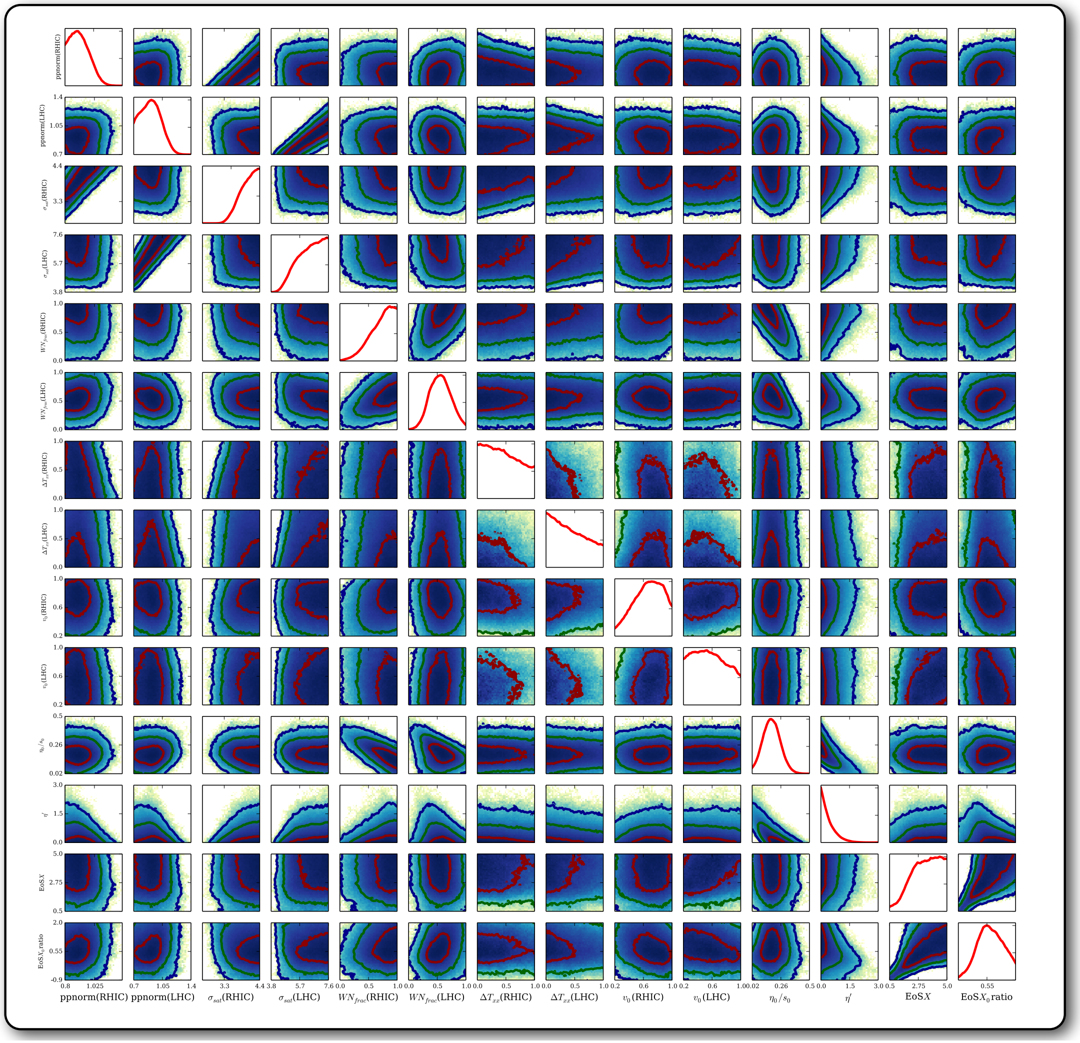
2 parameters for EoS, 2 for η/s

14x14
Posterior Likelihood

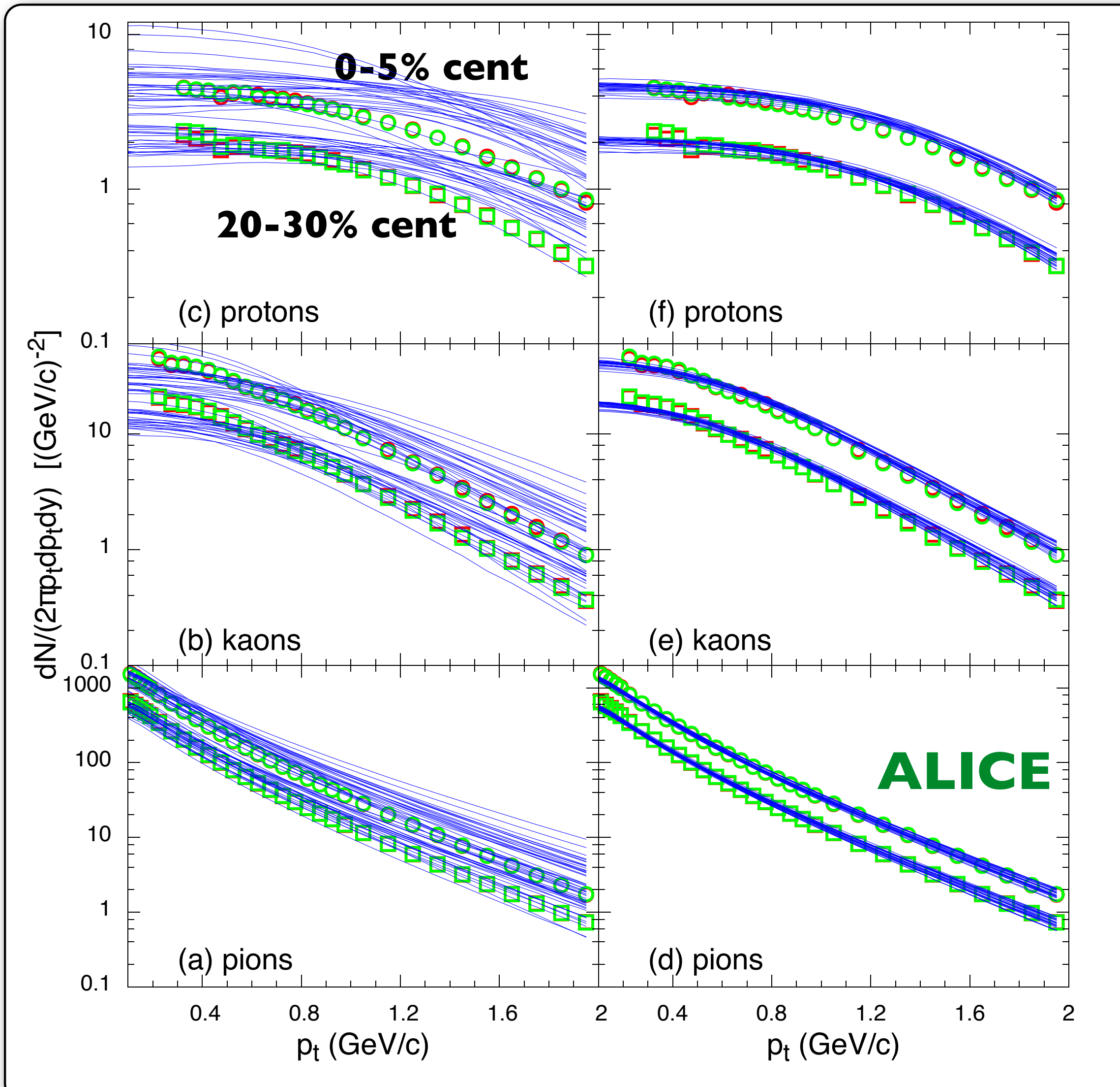
S.P., E.Sangaline, P.Sorensen & H.Wang, PRL 2015

RHIC Au+Au and LHC Pb+Pb Data

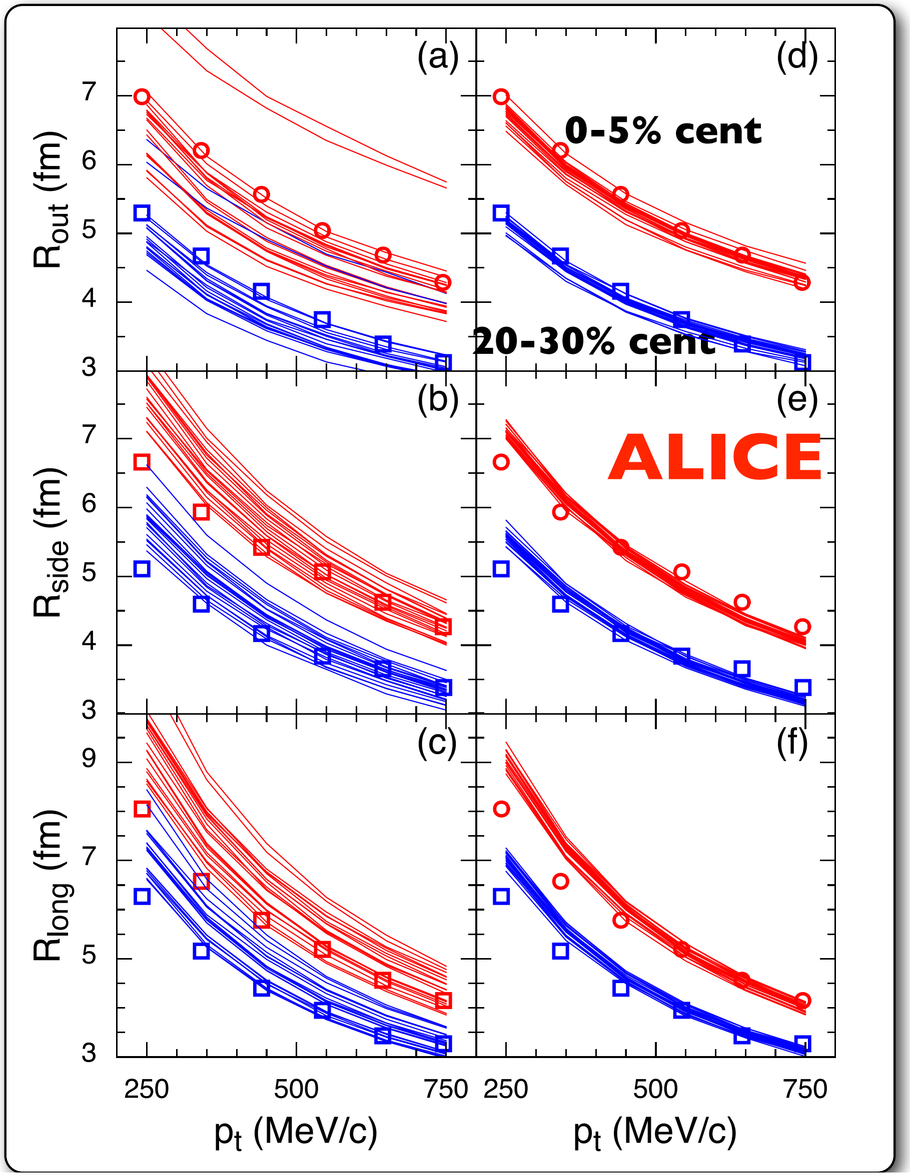
14 parameters, include Eq. of State



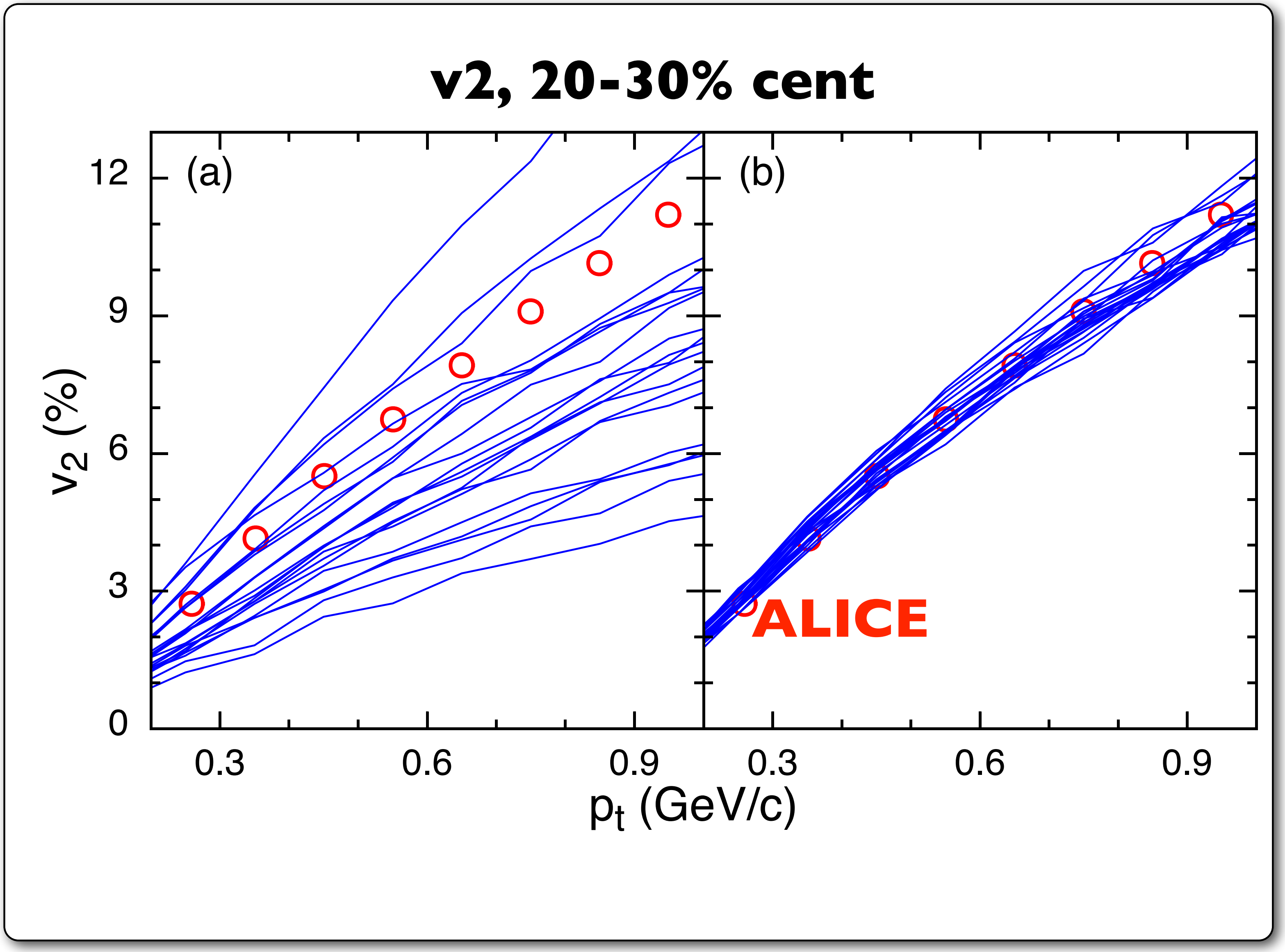
Sample Spectra from Prior and Posterior



Sample HBT from Prior and Posterior



Sample v_2 from Prior and Posterior

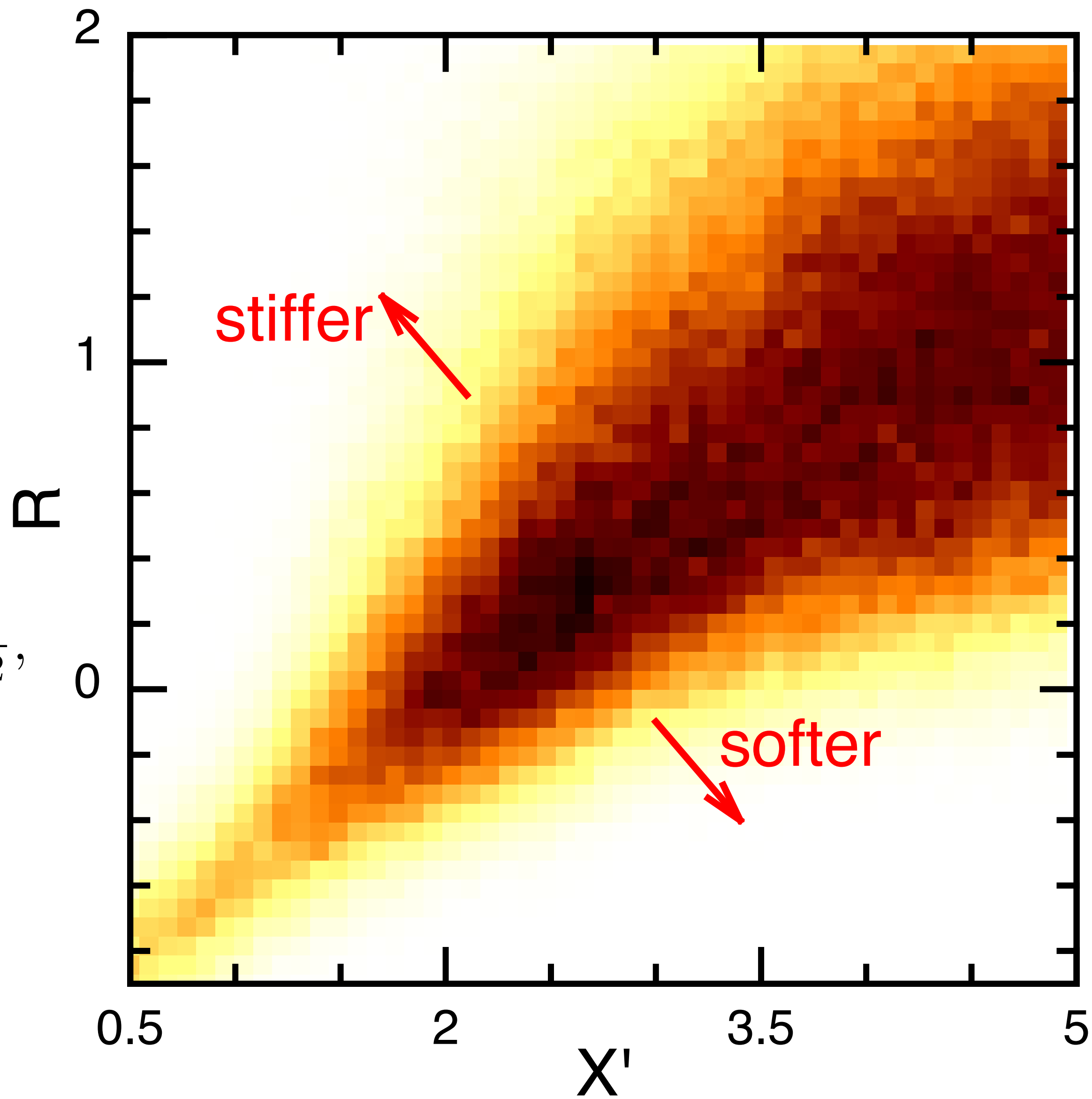


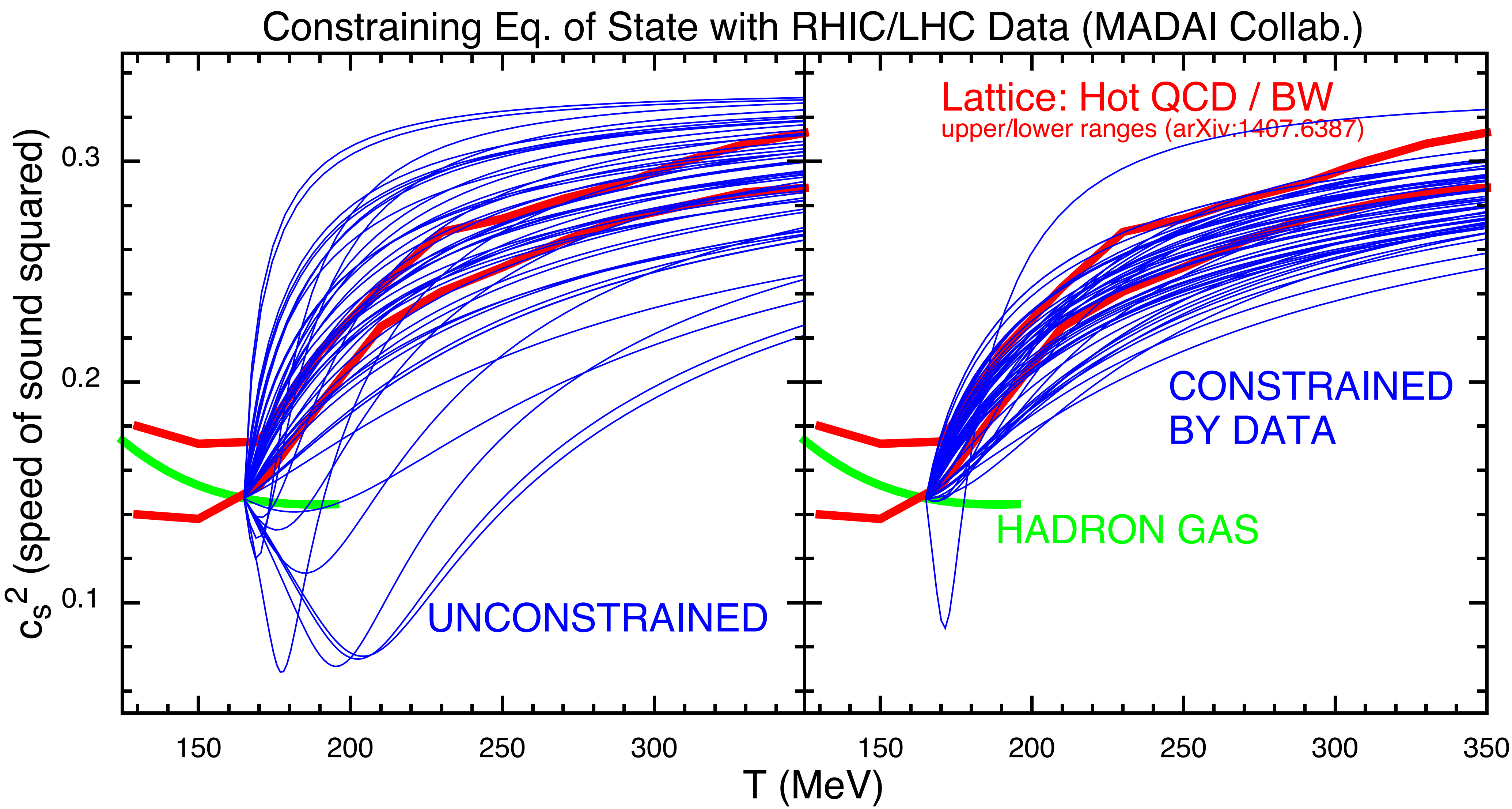
Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

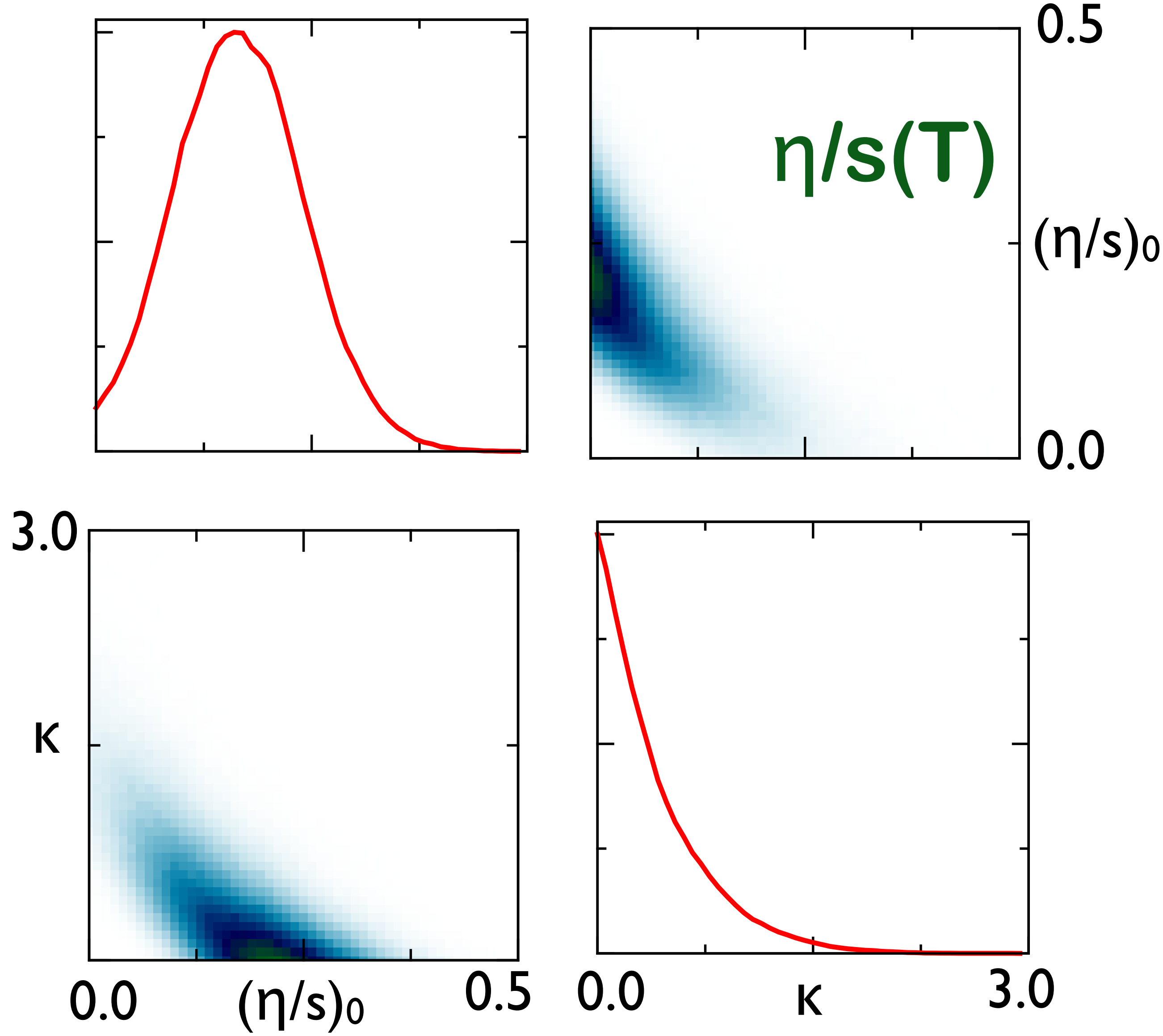
$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$




Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
UNCONSTRAINED
CONSTRAINED
BY DATA
HADRON GAS
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$

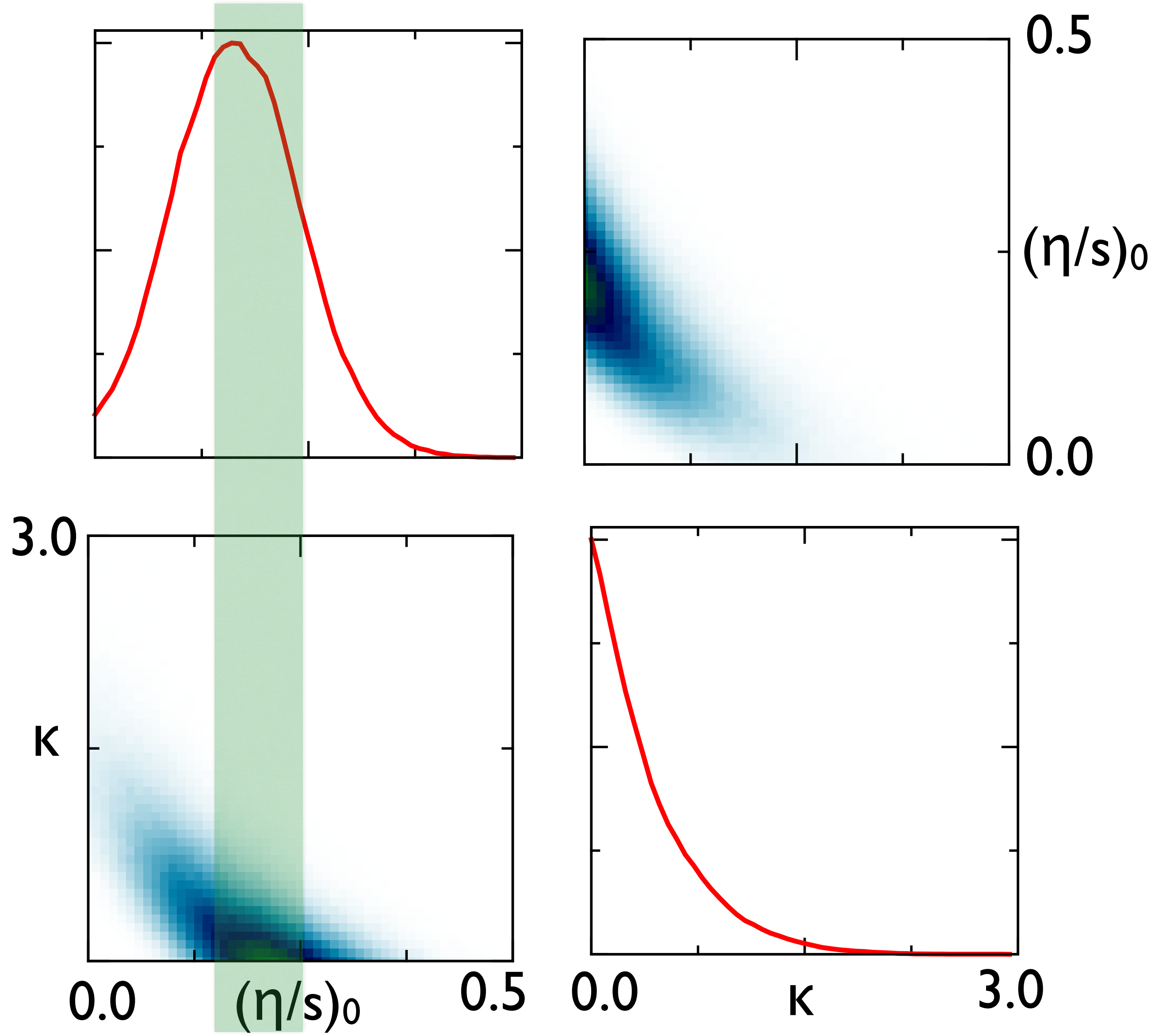


What should you expect for η/s at T=165 MeV?

- **ADS/CFT: 0.08**
- **Perturbative QCD: > 0.5 ($\sigma \approx$ 3 mb)**
- **Hadron Gas: $\approx$ 0.2 ($\sigma \approx$ 30 mb)**

Extracted η/s at T=165 MeV consistent with expectations for hadron gas!

Does not rise strongly in QGP



RESOLVING POWER OF OBSERVABLES

How does changing $y_{a,exp}$ or σ_a alter $\langle\langle x_i \rangle\rangle$ or $\langle\langle \delta x_i \delta x_j \rangle\rangle$?

We need $\frac{\partial}{\partial y_a^{(\mathrm{exp})}}\langle\langle x_i\rangle\rangle$ **NOT** $\frac{\partial}{\partial x_i} y_a^{(\mathrm{mod})}$

From covariances form MCMC trace + linear algebra….

E.Sangaline and S.P., arXiv 2015

RESOLVING POWER OF OBSERVABLES

$$\langle\langle x_i \rangle\rangle = \frac{\langle x_i \mathcal{L} \rangle}{\langle \mathcal{L} \rangle}$$

$$\frac{\partial}{\partial y_a^{(\exp)}} \langle\langle x_i \rangle\rangle = \langle\langle x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle - \langle\langle x_i \rangle\rangle \langle\langle (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle$$

$$= \langle\langle \delta x_i (\partial_a \mathcal{L})/\mathcal{L} \rangle\rangle$$

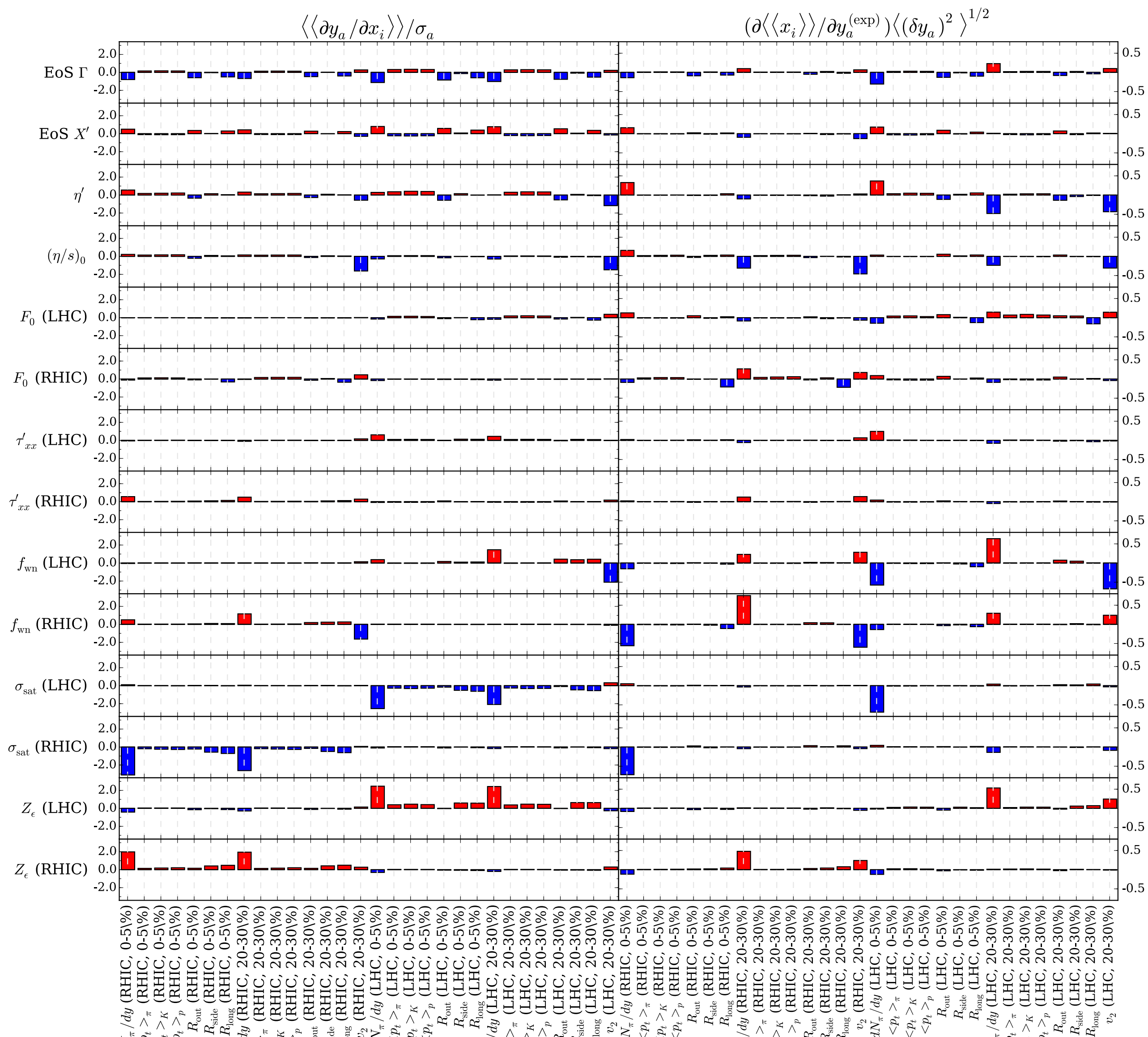
$$= -\Sigma_{ab}^{-1} \langle\langle \delta x_i \delta y_b \rangle\rangle \qquad \text{(for Gaussian)}$$

$$\delta x_i = x_i - \langle\langle x_i \rangle\rangle, \quad \delta y_a = y_a - y_a^{(\exp)}$$

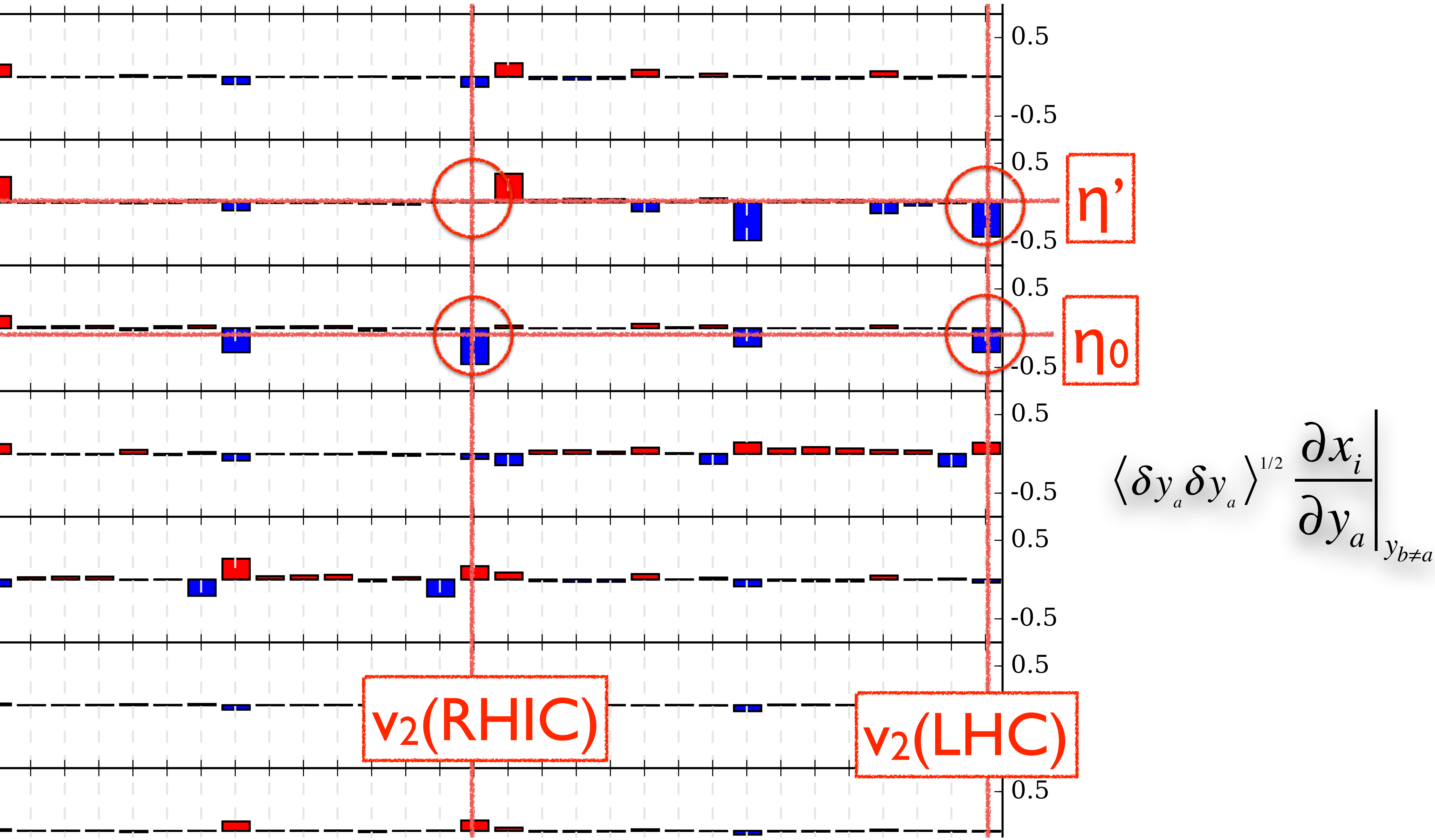
can find similar relation for $\frac{\partial}{\partial \sigma_a} \langle\langle \delta x_i \delta x_j \rangle\rangle$

E.Sangaline and S.P., PRC 2016

$$\frac{1}{\sigma_a}\frac{\partial y_a}{\partial x_i}\bigg|_{y_{b\neq a}}$$



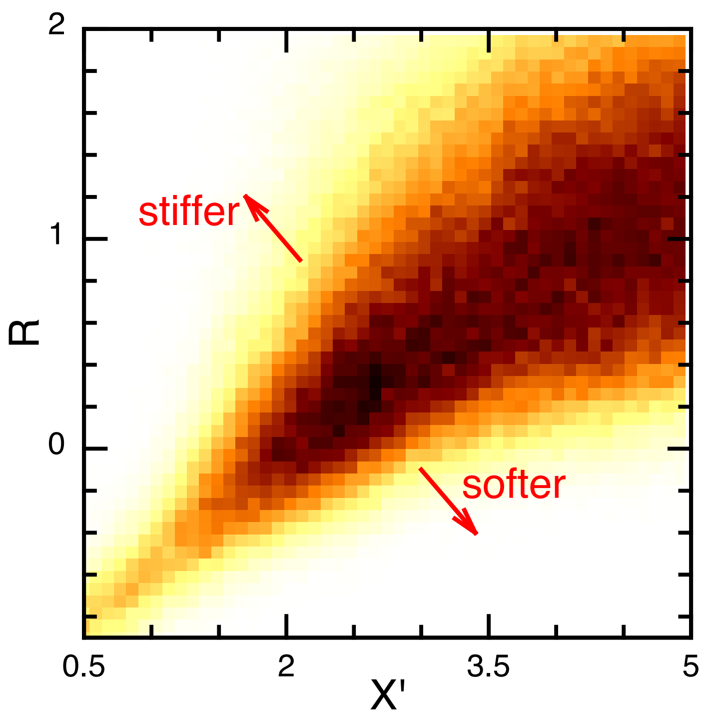
$$\langle\delta y_a\delta y_a\rangle^{1/2}\frac{\partial x_i}{\partial y_a}\bigg|_{y_{b\neq a}}$$

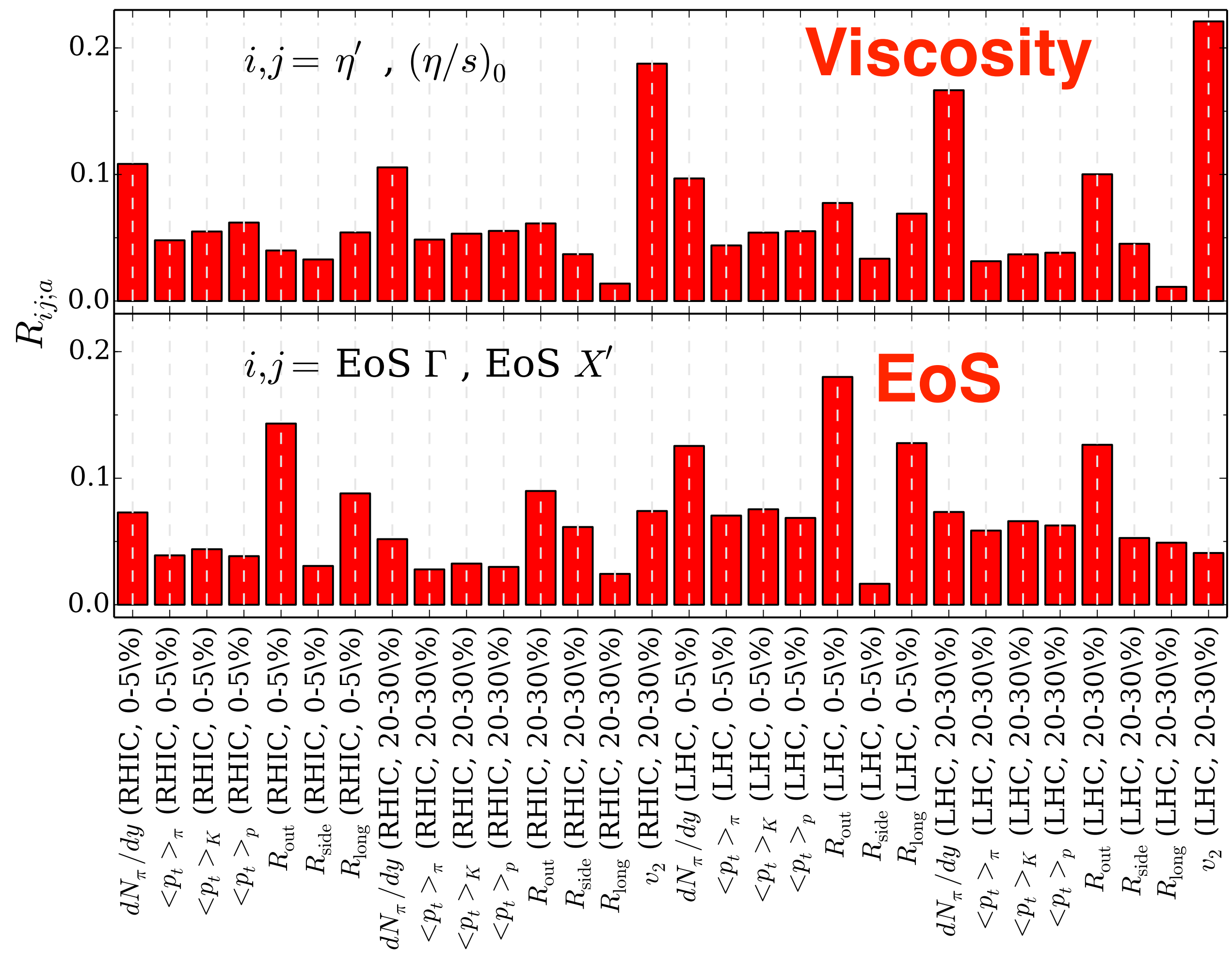


0.5
-0.5
η'
η0
v2(RHIC)
v2(LHC)
$\langle \delta y_a \delta y_a \rangle^{1/2} \left.\frac{\partial x_i}{\partial y_a}\right|_{y_{b\neq a}}$

2-Parameter Resolving Power

$$\frac{d}{d\sigma_y}\sqrt{\begin{vmatrix} \langle\langle \delta x_1 \delta x_1 \rangle\rangle & \langle\langle \delta x_1 \delta x_2 \rangle\rangle \\ \langle\langle \delta x_1 \delta x_2 \rangle\rangle & \langle\langle \delta x_2 \delta x_2 \rangle\rangle \end{vmatrix}}\langle \delta y \delta y \rangle^{1/2}$$





What determines EoS?

- Lots of observables
- Femtoscopic radii are important

What determines viscosity?

- Both v_2 and multiplicities
- T-dependence comes from LHC v_2

Validated collective wisdom of field

CONCLUSIONS

- Robust, emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from data
- Eq. of State consistent with lattice gauge theory
- Extends to other observables:
  diffusivity, jets, Eq. of state for $\mu_B \neq 0$
- Heavy-Ion Physics can be a Quantitative Science!!!!

Bayesian for Heavy-Ion Physics
Challenges Going Forward

1. Faithful representation of uncertainty
   — needs discussion
2. RHIC Beam Energy Scan
   — 3 D, more energies, include fluctuations
   — 1000s x more numerically expensive
3. Compare/Combine/Choose competing models